Propagation of Electromagnetic Waves in Rectangular Guides Filled With a Semiconductor in the Presence of a Transverse Magnetic Field

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Abstract—The characteristics of electromagnetic waves propagating in a semiconductor filled rectangular waveguide in the presence of a transverse magnetic field are studied. It is shown that only TE mode waves having y-independent field components (y being the direction of the steady magnetic field) and anomalous modes having all six field components can propagate. The propagation constant of waves characterized by a sinusoidal y dependence of fields is derived. Asymptotic expressions for the fields and the propagation constant are then obtained for the limiting case of a small external magnetic field and some recent experimental results are analyzed in this context.

I. Introduction

ELECTROMAGNETIC wave propagation in rectangular waveguides filled with magnetized ferrites have been extensively studied and are well discussed in the literature [1]. The permeability of magnetized ferrites being a tensor quantity, the modes of propagation are found to have features which are quite distinct from those found in the case of isotropic media. In addition to the TE modes, new types of modes like ferrite-guided and anomalous gyromagnetic modes are found to be possible.

The complex permittivity of a semiconductor in the presence of a magnetic field is also a tensor quantity. When a steady magnetic field is applied to a semiconductor-filled waveguide carrying electromagnetic waves, the electric fields perpendicular to the applied magnetic field become coupled. Hence, the characteristics of the waveguide modes are changed, as in the analogous case of magnetized ferrites where the microwave magnetic fields perpendicular to the steady magnetization are coupled. But, since the electric and magnetic fields are required to satisfy different conditions at the boundaries of the waveguide, the exact characteristics of the modes are likely to be different in the two cases. In recent years several workers [2]–[6] have studied experimentally the effect of a steady magnetic field on the propagation of electromagnetic waves in semiconductor filled waveguides. It is, therefore, of interest to examine the characteristics of the modes which are possible under these conditions. An analysis for rectangular waveguides and transverse magnetic fields has been made by Hirota [7] assuming that the conductivity of the semiconductor in a direction parallel to the magnetic field is very high, and, hence, that the electric field in this direction is equal to zero. In the present paper the same problem is studied, but the diagonal elements of the complex permittivity tensor are assumed to be equal. The method of analysis presented here is, however, applicable even if this assumption is not valid, but the algebraic manipulations in this case become very complicated.

In Section II, the characteristics of the modes are generally examined by considering Maxwell's equations and the boundary conditions. It is shown that in the present case no TM modes or TE modes other than those of the type TE_{n0} may be excited. In addition, anomalous mode waves having all the six field components are also possible. In Section III, the nature of propagation of the modes having a sinusoidal field intensity variation in the direction of the magnetic field are studied. It is found that the propagation characteristics are, in general, reciprocal. When the applied magnetic field is not too large, one may assume that the diagonal terms of the complex permittivity tensor are equal. Complete field solutions are given for this case in Section III. Section IV is devoted to the consideration of the solution for small magnetic fields, such that the off-diagonal elements of the permittivity tensor can be treated as small perturbations. By making asymptotic approximations, the general nature of the field distributions are illustrated. These results are discussed in relation to some recent experiments in Section V.

II. General Considerations

A rectangular waveguide as shown in Fig. 1 is assumed to be completely filled with a semiconductor. In the presence of an electromagnetic wave in the guide a dielectric and also a conduction current flow in the semiconductor. The dielectric current J_d is given by

$$J_d = \epsilon \frac{\partial E}{\partial t} \tag{1}$$

where ϵ is the permittivity of the semiconductor and E is the electric field vector. It has been suggested [8] that the dielectric current may be changed when a magnetic field is applied. However, this effect has not been experimentally observed, and even if it exists it

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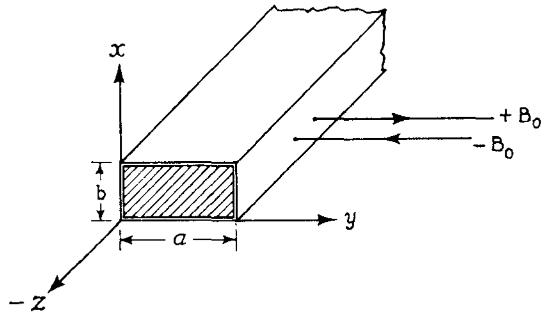

Fig. 1. Basic guide configuration.

would be small. It is assumed, therefore, that the dielectric current is unaffected by the magnetic field.

The conduction current in the absence of a magnetic field is

$$J_c = \sigma \boldsymbol{E}. \tag{2}$$

The signal frequency is assumed to be much less than the scattering frequency and, hence, σ is equal to the dc conductivity of the semiconductor. When a magnetic field is applied, the conduction current is modified due to the Hall effect. The Hall effect produces a field which is opposite to the microwave electric field [9]. In the case of a semiconductor with spherical energy surfaces the Hall field $\boldsymbol{E}_H$ is

$$\boldsymbol{E}_H = R_c \boldsymbol{B}_0 \times J_c \tag{3}$$

where R_c is the Hall coefficient and $\boldsymbol{B}_0$ is the steady magnetic field (assumed to be applied in the positive y direction). Hence, the modified conduction current is

$$J_{CH} = \sigma(\boldsymbol{E} - \boldsymbol{E}_H). \tag{4}$$

Now, assuming the time dependence of the electromagnetic fields to be $e^{j\omega t}$, (1), (3), and (4) may be combined and the total current J, written as

$$J = J_d + J_{CH} = [\epsilon] \frac{\partial \boldsymbol{E}}{\partial t} \tag{5}$$

where $[\epsilon]$, the complex permittivity tensor, is given by

$$[\epsilon] = \begin{bmatrix} \epsilon_1 & 0 & \epsilon_3 \\ 0 & \epsilon_2 & 0 \\ -\epsilon_3 & 0 & \epsilon_1 \end{bmatrix} \tag{6a}$$

and

$$\epsilon_1 = \epsilon \left[1 - \frac{j\sigma}{\omega\epsilon\{1 + (R_c B_0 \sigma)^2\}} \right]$$

$$\epsilon_2 = \epsilon \left[1 - \frac{j\sigma}{\omega\epsilon} \right]$$

$$\epsilon_3 = \frac{jR_c B_0 \sigma^2}{\omega\{1 + (R_c B_0 \sigma)^2\}} . \tag{6b}$$

Maxwell's equations in the case of media having a tensor permittivity are

$$\nabla \times \boldsymbol{H} = [\epsilon] \frac{\partial \boldsymbol{E}}{\partial t}$$

$$\nabla \times \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t} \tag{7}$$

where μ is the permeability of semiconductor.

Assuming as usual the z dependence of the field to be $e^{\Gamma z}$ (Γ being the propagation constant), one may obtain the following pair of equations in E_z and H_z from (7):

$$-D_y[j\omega\epsilon_3(\Gamma^2 + \omega^2\mu\epsilon_2) + j\omega\Gamma D_x(\epsilon_1 - \epsilon_2)]E_z + \left[\omega^2\mu\epsilon_2 \left\{\omega^2\mu\epsilon_1 + \Gamma^2\left(1 + \frac{\epsilon_1}{\epsilon_2}\right) + D_y^2 + \frac{\epsilon_1}{\epsilon_2} D_x^2\right\} + \Gamma^2(D_x^2 + D_y^2 + \Gamma^2)\right] H_z = 0 \tag{8}$$

$$D_y\left[j\omega\epsilon_1\left\{\omega^2\mu\epsilon_2 + \Gamma^2 + \frac{\epsilon_2}{\epsilon_1}(D_x^2 + D_y^2)\right\} - j\omega\epsilon_3\Gamma D_x\right] E_z + [\omega^2\mu\epsilon_3(\omega^2\mu\epsilon_2 + \Gamma^2 + D_x^2) + \Gamma D_x(\omega^2\mu\epsilon_2 + \Gamma^2 + D_x^2 + D_y^2)]H_z = 0 \tag{9}$$

where D_x and D_y stand respectively for ∂/∂_x and ∂/∂_y.

If we assume that $E_z = 0$ (TE modes), then (8) and (9) reduce to

$$\left[\Gamma^4 + \Gamma^2\left\{D_x^2 + D_y^2 + \omega^2\mu\epsilon_1\left(1 + \frac{\epsilon_2}{\epsilon_1}\right)\right\} + \omega^2\mu\epsilon_1\left(D_x^2 + \frac{\epsilon_2}{\epsilon_1} D_y^2 + \omega^2\mu\epsilon_2\right)\right] H_z = 0 \tag{10}$$

$$[\Gamma^3 D_x + \Gamma^2\omega^2\mu\epsilon_3 + \Gamma D_x(D_x^2 + D_y^2 + \omega^2\mu\epsilon_2) + \omega^2\mu\epsilon_3(D_x^2 + \omega^2\mu\epsilon_2)]H_z = 0. \tag{11}$$

Equations (10) and (11) cannot be simultaneously satisfied by the same value of Γ unless $D_y = 0$ or $H_z = 0$, even if $\epsilon_1 = \epsilon_2$. Hence, only those TE modes which have y-independent field components can propagate in the semiconductor filled guide in the presence of a magnetic field. It should, however, be noted that all these TE_{n0} modes which can propagate are characterized by having only the component of electric field along the y direction. Since the magnetic field is also applied in this direction there is no Hall field produced, and the characteristics of these modes should evidently remain unaltered in the presence of the magnetic field.

If we assume $H_z = 0$ (TM modes) (8) and (9) reduce to

$$-D_y[j\omega\epsilon_3(\Gamma^2 + \omega^2\mu\epsilon_2) + j\omega\Gamma D_x(\epsilon_1 - \epsilon_2)]E_z = 0 \tag{12}$$

$$D_y\left[j\omega\epsilon_1\left\{\omega^2\mu\epsilon_2 + \Gamma^2 + \frac{\epsilon_2}{\epsilon_1}(D_x^2 + D_y^2)\right\} - j\omega\epsilon_3\Gamma D_x\right] E_z = 0. \tag{13}$$

Equations (12) and (13) cannot be simultaneously satisfied, even if $\epsilon_1=\epsilon_2$ unless $D_y=0$ or $E_z=0$. However, if $D_y=0$, E_z must vanish everywhere since it must vanish at the boundaries. One may, hence, conclude that in a rectangular waveguide filled with a semiconductor, TM modes cannot propagate in the presence of a transverse magnetic field.

Equations (8) and (9) may, however, be satisfied by the same value of Γ if $H_z\neq 0$ and $E_z\neq 0$. Hence, in addition to the y-independent TE modes, the so-called anomalous modes having all the six field components would be possible.

III. Specific Solutions Assuming a Sinusoidal y-Dependence of the Fields

General solutions giving the field components of the anomalous modes are rather difficult to obtain. However, one may consider that the application of the magnetic field across the semiconductor-filled waveguide perturbs the modes usually possible in the guide. Hence, though all types of TE and TM modes are not possible there will be modes corresponding to the unperturbed TE or TM modes. The field components of such perturbed modes may be obtained by using our knowledge of the basic unperturbed modes. In this section we shall obtain the field components of the modes corresponding to the TE modes having a sinusoidal y-dependence of fields. The method followed here is similar to that used by Barzilai and Gerosa [10] in their analysis of propagation in rectangular waveguides filled with transversely magnetized ferrites.

Let a and b be the cross-sectional dimensions, parallel and perpendicular to the external magnetic field B_0, of the waveguide. E_z is assumed to have a y-dependence of the form $\sin(m\pi y/a)$ where m is an integer. Combining (8) and (9) and replacing D_y^2 by $-(m\pi/a)^2$ one obtains

$$[PD_x^4+QD_x^2+R]E_z=0 \tag{14}$$

where

$$P=\epsilon_1$$

$$Q=2\Gamma^2\epsilon_1+\omega^2\mu\epsilon_1^2+\omega^2\mu\epsilon_3^2+\omega^2\mu\epsilon_1\epsilon_2-(\epsilon_1+\epsilon_2)\frac{m^2\pi^2}{a^2}$$

$$R=\Gamma^4\epsilon_1+\Gamma^2\left\{\omega^2\mu\epsilon_1^2+\omega^2\mu\epsilon_3^2+\omega^2\mu\epsilon_1\epsilon_2-(\epsilon_1+\epsilon_2)\frac{m^2\pi^2}{a^2}\right\}+\left\{\omega^4\mu^2\epsilon_1^2\epsilon_2+\omega^4\mu^2\epsilon_2\epsilon_3^2-2\omega^2\mu\epsilon_1\epsilon_2\frac{m^2\pi^2}{a^2}+\epsilon_2\frac{m^4\pi^4}{a^4}\right\}.$$

It may be of interest to note here that propagation will, in general, be of a reciprocal nature. This is due to the absence in (14) of coefficients having terms that are functions of odd powers of Γ.

For values of B_0 such that $(R_cB_0\sigma)^2\ll 1$, the diagonal components of the complex permittivity tensor are all found to be equal, i.e., $\epsilon_1=\epsilon_2$. In the following analysis this assumption is introduced for the sake of simplicity. Under some experimental conditions the assumption may not be justified, but one may obtain results for these conditions by following the method of analysis given here retaining ϵ_1 and ϵ_2 as two different quantities.

Now, since $E_z\neq 0$, when $\epsilon_1=\epsilon_2$ (14) can be shown to reduce to

$$\left\langle D_x^4+D_x^2\left[2\left(\omega^2\mu\epsilon_1+\Gamma^2-\frac{m^2\pi^2}{a^2}\right)+\omega^2\mu\epsilon_3\frac{\epsilon_3}{\epsilon_1}\right]+\left[\left(\omega^2\mu\epsilon_1+\Gamma^2-\frac{m^2\pi^2}{a^2}\right)^2+\omega^2\mu\epsilon_3\frac{\epsilon_3}{\epsilon_1}(\Gamma^2+\omega^2\mu\epsilon_1)\right]\right\rangle=0. \tag{15}$$

The four roots $\gamma_{1,2}^2$ of (15) are given by

$$\gamma_{1,2}^2=-\left[\omega^2\mu\epsilon_1+\Gamma^2-\frac{m^2\pi^2}{a^2}\right]-\frac{1}{2}\omega^2\mu\epsilon_3\frac{\epsilon_3}{\epsilon_1}\pm\frac{1}{2}\left[\left(\omega^2\mu\epsilon_3\frac{\epsilon_3}{\epsilon_1}\right)^2-4\omega^2\mu\epsilon_3\frac{\epsilon_3}{\epsilon_1}\frac{m^2\pi^2}{a^2}\right]^{1/2}. \tag{16}$$

Mikaelian [11] has shown that the boundary conditions at $x=0$, $x=b$ cannot be satisfied by either of the two independent birefringent modes corresponding to the four roots $\pm\gamma_{1,2}$, but that a linear combination of these must be used. That is,

$$E_z=(Ae^{\gamma_1x}+Be^{-\gamma_1x}+Ce^{\gamma_2x}+De^{-\gamma_2x})\cdot\sin\left(\frac{m\pi y}{a}\right)e^{(j\omega t+\Gamma z)} \tag{17}$$

where A, B, C, and D are four arbitrary constants. Substitution of (17) in Maxwell's equations gives

$$E_y=\left(\frac{a}{m\pi}\right)\frac{\epsilon_1}{\epsilon_3(\Gamma^2+\omega^2\mu\epsilon_1)}\cdot\left[Ae^{\gamma_1x}\left\{\gamma_1(a'+b')+\frac{\Gamma\epsilon_3}{\epsilon_1}\frac{m^2\pi^2}{a^2}\right\}+Be^{-\gamma_1x}\left\{-\gamma_1(a'+b')+\frac{\Gamma\epsilon_3}{\epsilon_1}\frac{m^2\pi^2}{a^2}\right\}+Ce^{\gamma_2x}\left\{\gamma_2(a'-b')+\frac{\Gamma\epsilon_3}{\epsilon_1}\frac{m^2\pi^2}{a^2}\right\}+De^{-\gamma_2x}\left\{-\gamma_2(a'-b')+\frac{\Gamma\epsilon_3}{\epsilon_1}\frac{m^2\pi^2}{a^2}\right\}\right]\cdot\cos\left(\frac{m\pi y}{a}\right)e^{(j\omega t+\Gamma z)} \tag{18}$$

and,

$$E_x = \frac{\epsilon_1}{\epsilon_3(\Gamma^2 + \omega^2\mu\epsilon_1)}\left[Ae^{\gamma_1 x}\left(\frac{\Gamma\epsilon_3}{\epsilon_1}\gamma_1 - a' + b'\right) + Be^{-\gamma_1 x}\left(-\frac{\Gamma\epsilon_3}{\epsilon_1}\gamma_1 - a' + b'\right) + Ce^{\gamma_2 x}\left(\frac{\Gamma\epsilon_3}{\epsilon_1}\gamma_2 - a' - b'\right) + De^{-\gamma_2 x}\left(-\frac{\Gamma\epsilon_3}{\epsilon_1}\gamma_2 - a' - b'\right)\right] \cdot \sin\left(\frac{m\pi y}{a}\right)e^{(j\omega t + \Gamma z)} \tag{19}$$

where

$$a' = \frac{1}{2}\omega^2\mu\epsilon_3\frac{\epsilon_3}{\epsilon_1};$$

$$b' = \frac{1}{2}\left[\left(\omega^2\mu\epsilon_3\frac{\epsilon_3}{\epsilon_1}\right)^2 - 4\omega^2\mu\frac{\epsilon_3^2}{\epsilon_1}\frac{m^2\pi^2}{a^2}\right]^{1/2}.$$

The boundary conditions demand that

$$E_z = 0 \quad \text{at } x = 0, b$$
$$E_y = 0 \quad \text{at } x = 0, b.$$

Implementation of the four boundary conditions results in a system of four linear homogenous equations in A, B, C, and D. For the nontrivial case the determinant of the coefficients of this set of equations must vanish. On expansion this determinantal equation reduces to the transcendental equation

$$2\gamma_1\gamma_2(a'^2 - b'^2)(1 - \cosh\gamma_1 b\cosh\gamma_2 b) = -\left\{\gamma_1^2(a' + b')^2 + \gamma_2^2(a' - b')^2\right\}\sinh\gamma_1 b\sinh\gamma_2 b. \tag{20}$$

The solution of (20) yields the required value of the propagation constant Γ. The coefficients B, C, and D may now be evaluated in terms of A. On evaluation one obtains

$$B = A\,\frac{2\gamma_1 e^{\gamma_1 b} - (\gamma_1 + \gamma_2)e^{\gamma_2 b} + (\gamma_2 - \gamma_1)e^{-\gamma_2 b}}{\Delta} \tag{21}$$

$$C = A\,\frac{(\gamma_2 - \gamma_1)e^{\gamma_1 b} + (\gamma_2 + \gamma_1)e^{-\gamma_1 b} - 2\gamma_2 e^{-\gamma_2 b}}{\Delta} \tag{22}$$

$$D = A\,\frac{-(\gamma_1 + \gamma_2)e^{\gamma_1 b} - (\gamma_2 - \gamma_1)e^{-\gamma_1 b} + 2\gamma_2 e^{\gamma_2 b}}{\Delta} \tag{23}$$

where

$$\Delta = -2\gamma_1 e^{-\gamma_1 b} + (\gamma_1 - \gamma_2)e^{\gamma_2 b} + (\gamma_2 + \gamma_1)e^{-\gamma_2 b}. \tag{24}$$

The roots $\gamma_{1,2}$ are now known quantities whose values can be determined from (16) and (20). As the coefficients B, C, and D have been obtained in terms of A, all the field components can be expressed in terms of the excitation represented by A. Since these expressions are rather involved, and since solutions of (20) may be obtained only numerically, it is difficult to examine the characteristics of the field distribution in the general case.

However, it may be noted that if B_0 be small, ϵ_3 is a small quantity and the expressions for the fields may be simplified. The propagation characteristics for the waveguide when this assumption is valid are discussed in the following section.

IV. Approximation for Small Magnetic Field

The expression for the propagation constant derived in the earlier section must reduce to that obtainable in the isotropic medium case when the external magnetic field $B_0 = 0$. Since in this case

$$\Gamma_0^2 = -\left(\omega^2\mu\epsilon_1 - \frac{m^2\pi^2}{a^2}\right) \tag{25}$$

one finds from (16) that

$$\gamma_{1,2} = 0 \tag{26}$$

where Γ_0 is the unperturbed propagation constant in the semiconductor-filled guide in the absence of any transverse external magnetic field.

Equation (16) may be written as

$$\gamma_{1,2}^2 = \delta\Gamma - a' \pm b' \tag{27}$$

where

$$\delta\Gamma = -\left(\Gamma^2 + \omega^2\mu\epsilon_1 - \frac{m^2\pi^2}{a^2}\right). \tag{28}$$

If it is assumed that $(\gamma_{1,2}b) \ll 1$ one can approximately write

$$\sinh\gamma_{1,2}b = \gamma_{1,2}b \quad \text{and} \quad \cosh\gamma_{1,2}b = 1 + \frac{(\gamma_{1,2}b)^2}{2}.$$

On substitution in (20) it turns out that

$$\delta\Gamma = 0 \tag{29}$$

$$\gamma_{1,2}^2 = -a' \pm b. \tag{30}$$

It should be noted that the assumption $(\gamma_{1,2}b) \ll 1$ is more restrictive than the earlier assumption $(R_cB_0\sigma)^2 \ll 1$. This may be shown by considering the value of $\sqrt{a'}b$ which is the order of $\gamma_{1,2}b$. Putting $b = 10^{-2}$ m (frequencies in the 3 cm band) and the values of other parameters one obtains

$$\sqrt{a'}\,b = 2.7\frac{\sigma}{\sqrt{\epsilon_r}}R_cB_0\sigma \qquad \text{for } \frac{\sigma}{\omega\epsilon} \ll 1$$

$$\sqrt{a'}\,b = 2.7\sqrt{\sigma\omega\epsilon_0}\,R_cB_0\sigma \qquad \text{for } \frac{\sigma}{\omega\epsilon} \gg 1$$

where ϵ_0 is the free space permittivity and ϵ_r the dielectric constant of the semiconductor. It is, thus, seen that the assumption $(\gamma_{1,2}b) \ll 1$ requires that $(R_cB_0\sigma)$ multiplied by a quantity greater than unity, for the usual values of σ, should be small in comparison to one.

It is found from (29) that for small values of B_0 the propagation constant is equal to its unperturbed value.

However, the fields are not the same as those of the isotropic case. One may write down the varying components of the electromagnetic fields in the guide, using the simplified expressions for B, C, and D obtained by substituting (27) through (30) into (21) through (24). One thus obtains

$$E_x = A' \frac{\epsilon_1}{\epsilon_3}\left(\frac{a}{m\pi}\right)^2\left(2 - \frac{\Gamma\epsilon_3}{\epsilon_1}b\right) \cdot \left[1 + \frac{\frac{\Gamma\epsilon_3}{\epsilon_1} + a'b}{1 - \frac{\Gamma\epsilon_3}{2\epsilon_1}b}x - a'x^2\right] \cdot \sin\left(\frac{m\pi y}{a}\right)e^{(j\omega t+\Gamma z)}$$

$$E_y = A'\left(\frac{\Gamma a}{m\pi}\right)x(x-b)\cos\left(\frac{m\pi y}{a}\right)e^{(j\omega t+\Gamma z)}$$

$$E_z = A'x(x-b)\sin\left(\frac{m\pi y}{a}\right)e^{(j\omega t+\Gamma z)} \tag{31a}$$

where

$$A' = \frac{4Ab'}{2-\gamma_1 b}\cdot$$

Substitution of (31a) into the second of Maxwell's equation (7) enables us to work out the corresponding components of the varying magnetic field as

$$H_x = jA'\omega\epsilon_1\left(\frac{a}{m\pi}\right)x(x-b)\cos\left(\frac{m\pi y}{a}\right)e^{(j\omega t+\Gamma z)}$$

$$H_y = \frac{jA'}{\omega\mu}\left(\frac{a}{m\pi}\right)^2\left[2\Gamma\frac{\epsilon_1}{\epsilon_3} - \omega^2\mu\epsilon_1(2x-b)\right] \cdot \sin\left(\frac{m\pi y}{a}\right)e^{(j\omega t+\Gamma z)}$$

$$H_z = -\frac{2jA'}{\omega\mu}\frac{\epsilon_1}{\epsilon_3}\left(\frac{a}{m\pi}\right)\cos\left(\frac{m\pi y}{a}\right)e^{(j\pi t+\Gamma z)}. \tag{31b}$$

Equations (31a) and (31b), as expected, are seen to reduce to those of the TE_{0m} mode when $B_0=0$. Furthermore, (31a) and (31b) point to an asymmetric x dependence of the fields. This point is further discussed in the next section where certain experimental results are analyzed in view of the theory developed in this section.

V. Discussion

Several experiments have been performed in recent years on the propagation of microwaves through semiconductors in the presence of external magnetic fields. The experiments using longitudinal magnetic fields and cylindrical structures are not relevant to the theory developed here. However, the results of the experiment of Barlow and Koike [4] may be explained by considering the field distribution derived in Section IV. Their arrangement using TE_{01} incident waves is shown in Fig. 2. The results indicated that the structure was nonreciprocal in nature, and that the propagation characteristics for the oppositely polarized circular waves, which are set up inside the guide as a result of B_0, were different for opposite directions of either the magnetic field, or the wave propagation, or the type of conductivity of the semiconductor material used.

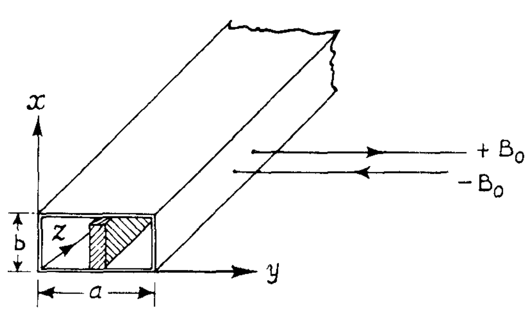

Fig. 2. Experimental arrangement of Barlow and Koike.

The following are the data related to the experiment:

$f=9975$ Mc/s.
n-type Ge $\sigma=6.25$ V/m.
$\epsilon_r=16$, $a=2.28$ cm $=0.0228$ m, $b=1.015$ cm $=0.01015$ m.
$B_0=0.3$ Wb/m^2.
$\mu_H=R_c\sigma=0.425$ m²/volt (see [12]).

It is seen that with these parameter values

$$(R_cB_0\sigma)^2 = (0.3\times 0.425)^2 = 0.016.$$

This can be considered negligible in comparison with unity. Hence, the assumption of equal diagonal elements in the complex permittivity tensor holds good in this case.

Our calculations indicate that though $\gamma_1 b$ or $\gamma_2 b$ are less than unity, the condition $(\gamma_{1,2}b)\ll 1$ is not strictly satisfied in this case. The expressions for the field given later, therefore, involve some error. But, the qualitative nature of the variation of the field, the subject of discussion in this section, should not be altered due to these errors.

Making appropriate calculations it is seen from (31) that,

$$E_x = A'\frac{\epsilon_1}{\epsilon_3}\left(\frac{a}{m\pi}\right)^2\left(2-\frac{\Gamma\epsilon_3}{\epsilon_1}b\right) \cdot [1 - 64.9993x + 18.8309\times 10^2x^2 + j(-20.2118x + 13.2744\times 10^2x^2)] \cdot \sin\left(\frac{\pi y}{a}\right)e^{(j\omega t+\Gamma z)}. \tag{32}$$

The real and imaginary parts and also the absolute magnitude of E_x as obtained from (32) by assuming the multiplying constant to be unity, are plotted in Fig. 3 for different values of x. It is seen that the magnitude of

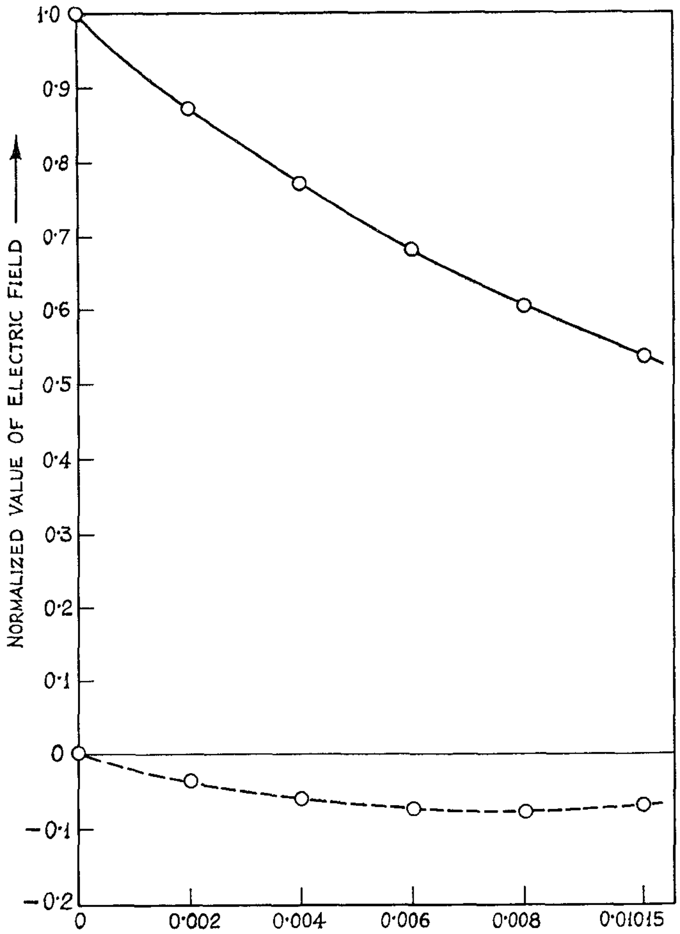

Fig. 3. Plot of real and imaginary parts, and the absolute magnitude of E_x vs. x. The straight line is the real part and absolute magnitude of E_x. The dotted line is the imaginary part of E_x.

E_x uniformly decreases with increase in x for the assumed direction of the magnetic field. It is then evident that for the reverse direction of magnetic field the magnitude of E_x would increase with increase in x. This is also true for the other components of the electric field. This result provides an explanation for the nonreciprocal nature of propagation observed in Barlow and Koike's experiment. In this experiment a semiconductor sample filling partially the narrow and also the broad sides of the waveguide was used. Hence, the magnitude of the electric field in the semiconductor is larger for one direction of the magnetic field than for the opposite direction. Since the attenuation is mostly introduced by the semiconductor sample, evidently it would be larger for that direction of the magnetic field for which the magnitude of the electric field in the sample is larger. It is also evident that since the electric field is maximum at $y=a/2$, this nonreciprocal effect would be more prominent when the sample is placed at the centre of the broad dimension. It should however be noted that this explanation is only qualitative in character, and a quantitative analysis of the problem would involve matching of the fields at the various free-space semiconductor interfaces.

VI. Conclusions

It has been shown that in a semiconductor-filled rectangular waveguide pure TM and TE mode waves except the TE_{n0} types cannot propagate in the presence of a transverse magnetic field. Anomalous modes having all the six field components are, however, possible. Propagation is, in general, reciprocal. The field patterns of the anomalous mode corresponding to the TE_{01} mode in the isotropic medium filled guide, are found to be asymmetrically distorted. This asymmetric distortion provides an explanation for the nonreciprocal propagation observed by Barlow and Koike [4].

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